

# Distributed Backup Placement in WSNs and Planar Graphs

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## Introduction

- The **backup-placement problem** was introduced by Halldórsson et al. in 2015. This problem turned out to be **very challenging** in general networks.
- We focus on wireless networks, specifically looking into solutions that are **significantly better than polynomial (and even than linear) solutions**.
- Scenario example: several nodes in a network have packages whose backups (or the package itself) need to find a placement elsewhere in the network, due to overload in these nodes areas, **in order to improve fault-tolerance and data integrity**.
- Because of the lack of capacity to store or process the data on the node itself, it is mandatory, when the local memory is full, to **find a backup-placement to the data outside of the node**. The backup-placement problem is defined as follows:
  - How to place the data **only once** in a safe and stable node in order to assure with a high degree of certainty the data integrity and minimization of the network load.
  - How to do so **without creating an additional data overflow** on other areas of the network.

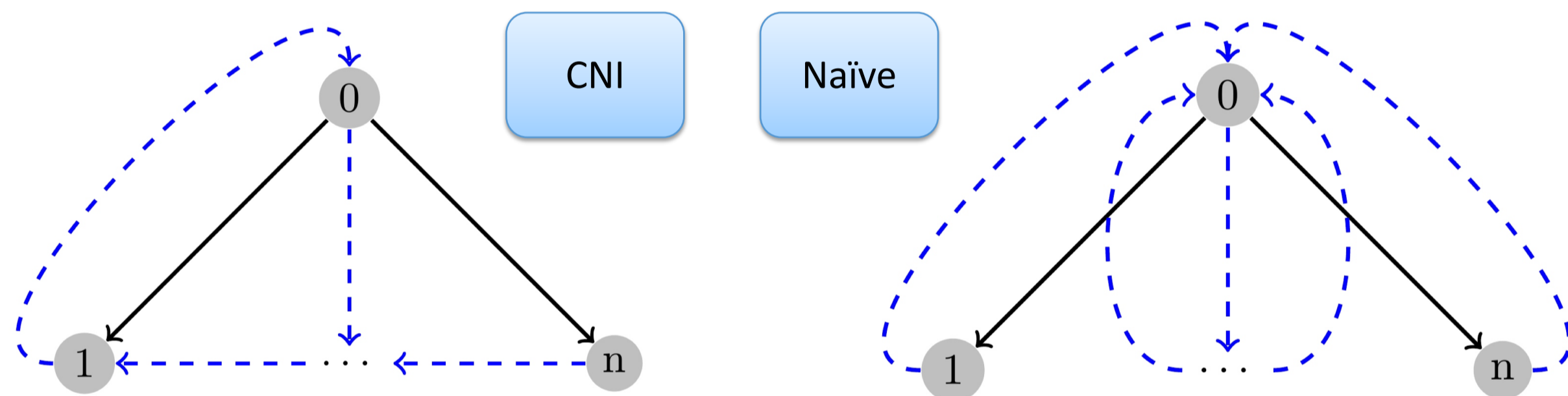
## Backup placement in trees

- The procedure receives a tree  $T = (V, E)$  as input :
  - Algorithm 1** is the naive one, which computes an  $O(1)$ -backup placement of  $T$ .
  - Algorithm 2** is based on constant neighborhood, with  $O(1)$  time complexity.

### Algorithm 1 Naive Distributed Tree Backup Placement Algorithm in $O(1)$

```

1: procedure NAIVE-TREE-BP(NODE  $v \in T$ , TREE  $T$ )
2:   if  $v$  is not a leaf then
3:      $v.BP \leftarrow \text{Arbitrary}(v.children)$ .
4:   else
5:      $v.BP \leftarrow v.Parent$ 
    
```

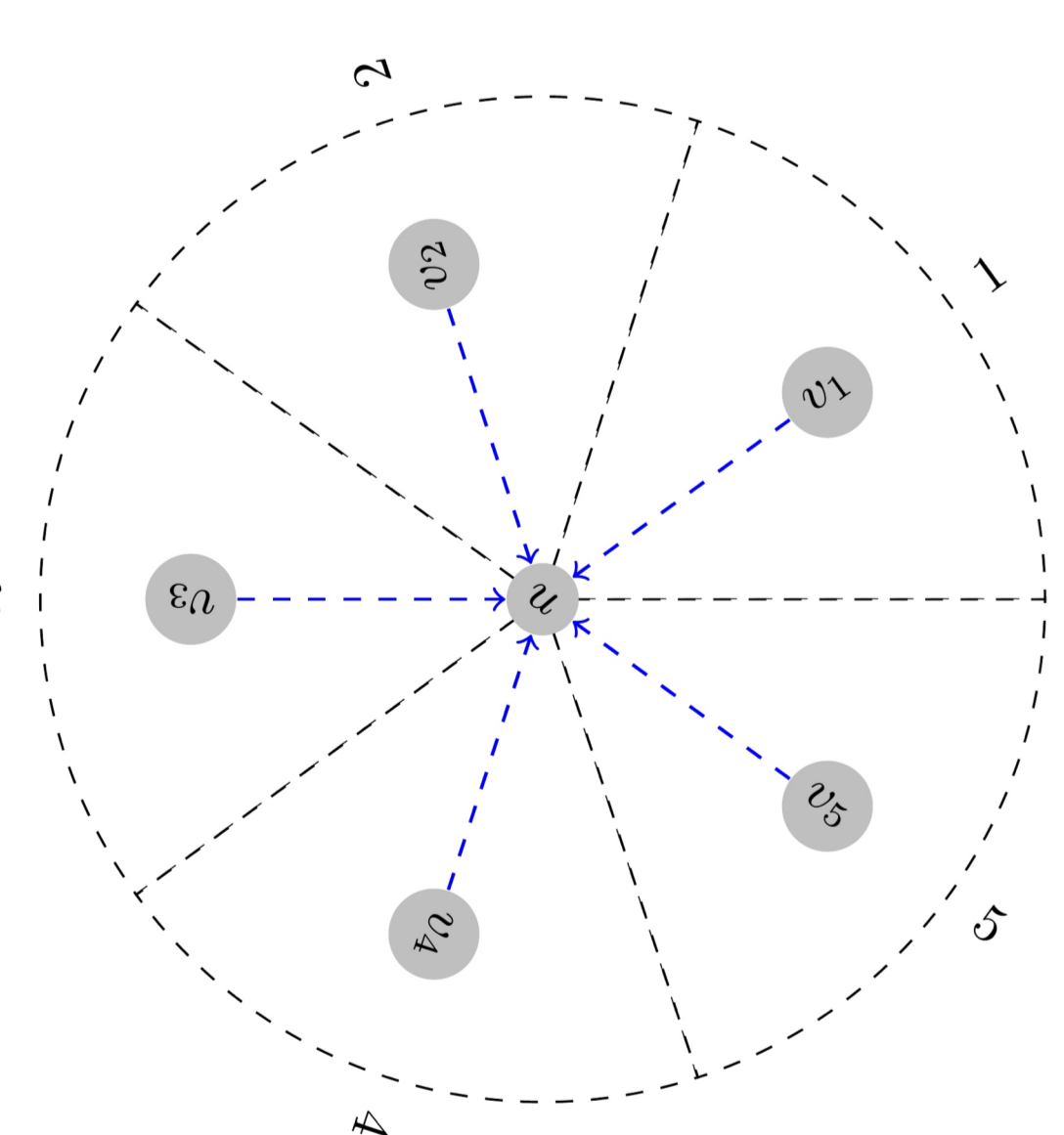


### Algorithm 2 Constant Neighborhood Independence Distributed Tree Backup Placement Algorithm in $O(1)$

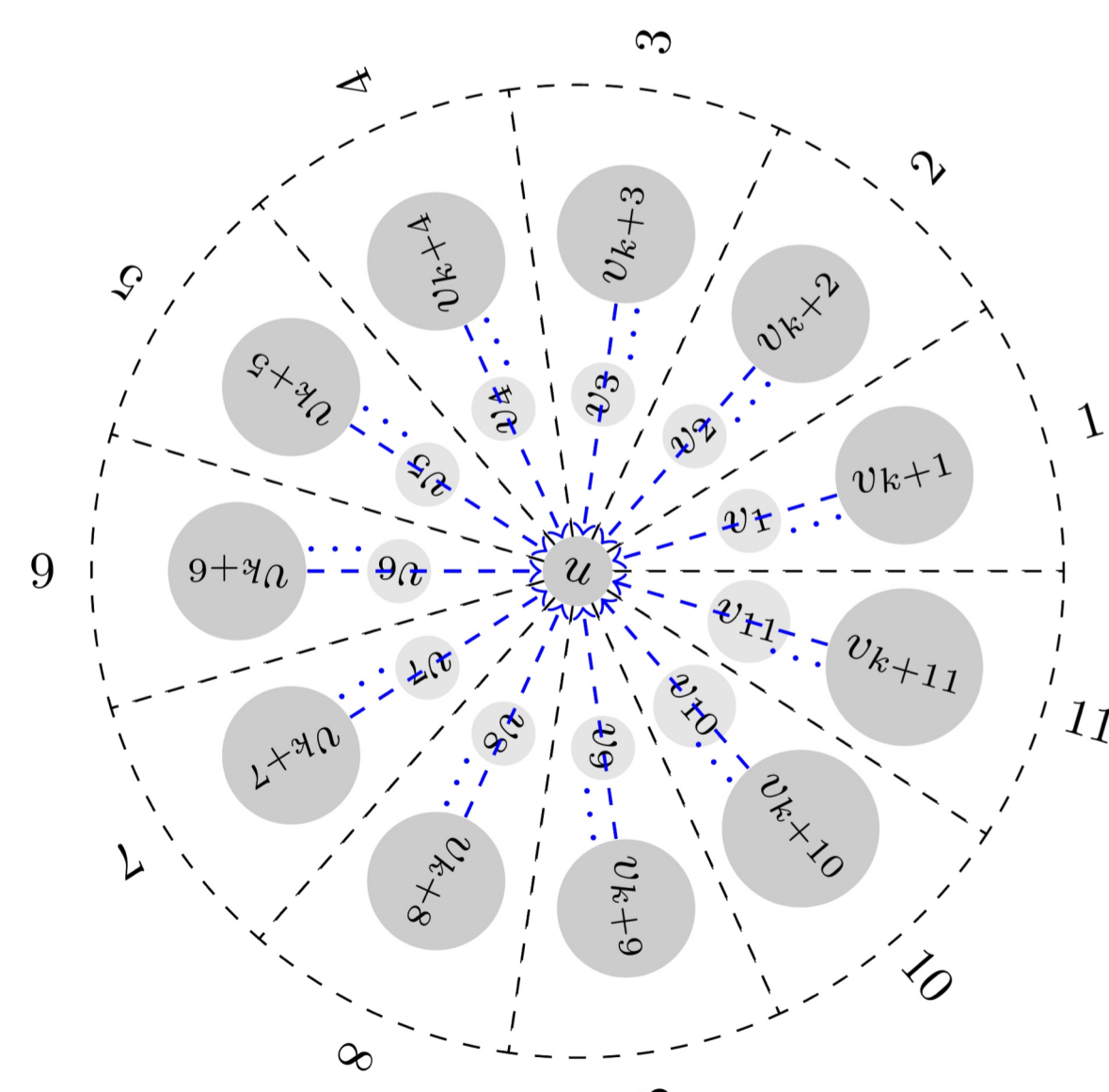
```

1: procedure CNI-TREE-BP(NODE  $u$ , GRAPH  $G$ ), SUBGRAPH  $T$ 
2:   if  $v$  is not a leaf then
3:      $v.BP \leftarrow \text{Arbitrary}(v.children)$ .
4:   else if  $\exists w$  sibling of  $v$  ( $ID(w) < ID(v)$ )  $\wedge \nexists z$  sibling of  $v$  ( $ID(w) < ID(z) < ID(v)$ )  $\wedge (v, w), (w, z) \in E(G)$  then
5:      $v.BP \leftarrow w$ .
6:   else
7:      $v.BP \leftarrow v.Parent$ .
    
```

- After we proved that each vertex of  $G$  is selected by at most  $c$  vertices of  $T$  if  $G$  is a graph with neighborhood independence of at most  $c$ , and  $T$  is a subtree of  $G$ , we can now prove that **in the case of wireless networks the parameter  $c$  is a constant**, based on the properties of the  $UDG$  in case of a homogeneous network  $c=5$ .
- We can also prove that even in case of a **heterogeneous network**, i.e. a network in which all nodes radii are different, based on the properties of the bounded disk graph ( $BDG$ ), **parameter  $c$  is small:  $c=11\log(R_{max}/R_{min})$** .



**Unit Disk Graph model**  
 A root node  $v_0$ , which forms 5 different cliques, and  $\{v_1, \dots, v_5\}$  nodes which must choose  $v_0$  as their backup placement dest'



**Bounded Disk Graph model**  
 A root node  $v_0$ , which forms  $c=11\log(R_{max}/R_{min})$  different cliques

## Backup placement in forests

- We devise a procedure for computing  $O(1)$ -**backup placement in forest**. We assume that each vertex do not knows its parent, nor to which tree in the forest it belongs. The procedure receives a forest  $F = (V, E)$  as input and proceeds as follows.
- An exemplification of the Distributed Tree Discovery are in **red** and the Forest Backup Placement are in **blue**.

### Algorithm 3 The Distributed Tree Discovery Algorithm

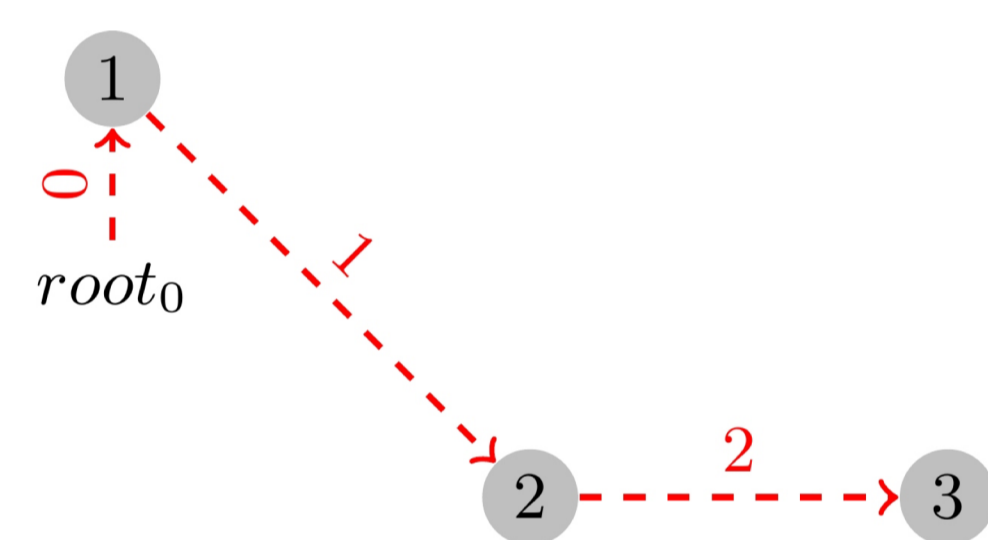
```

1: procedure D-TREE-DISCOVERY(NODE  $v$ , GRAPH  $G$ , BEACON  $B$ )
2:   ASSERT:  $B = \{ID_{Parent}, ID_{Source}, ID_{root}, d_{root}\}$ 
3:   if  $v.d_{root} > d_{root} + 1$  then
4:     ASSERT: The neighbor of  $v$ , from which the beacon has been received, holds a shorter path to root
5:      $v.parent \leftarrow ID_{Source}$ 
6:      $v.d_{root} \leftarrow d_{root} + 1$ 
    
```

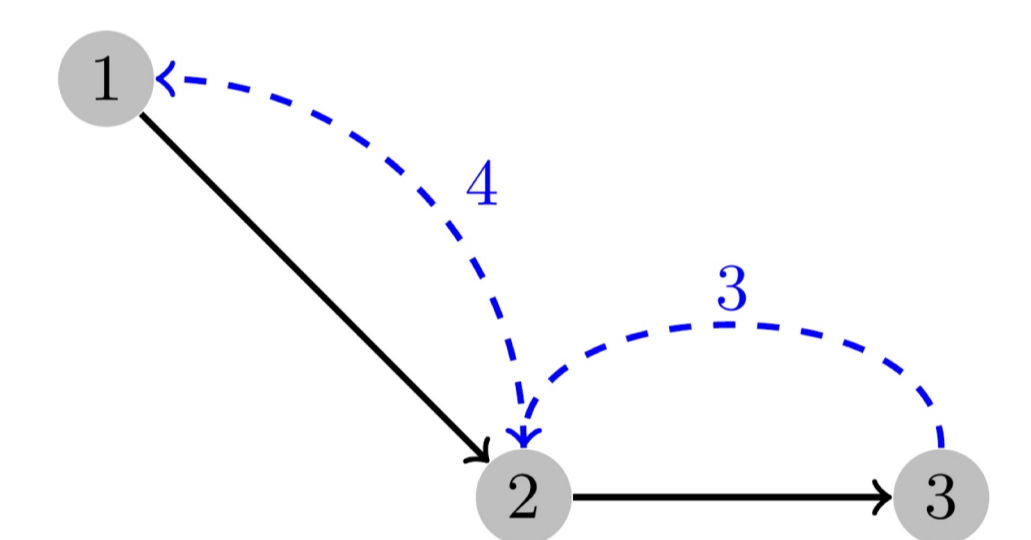
### Algorithm 4 The Forest Backup Placement Algorithm

```

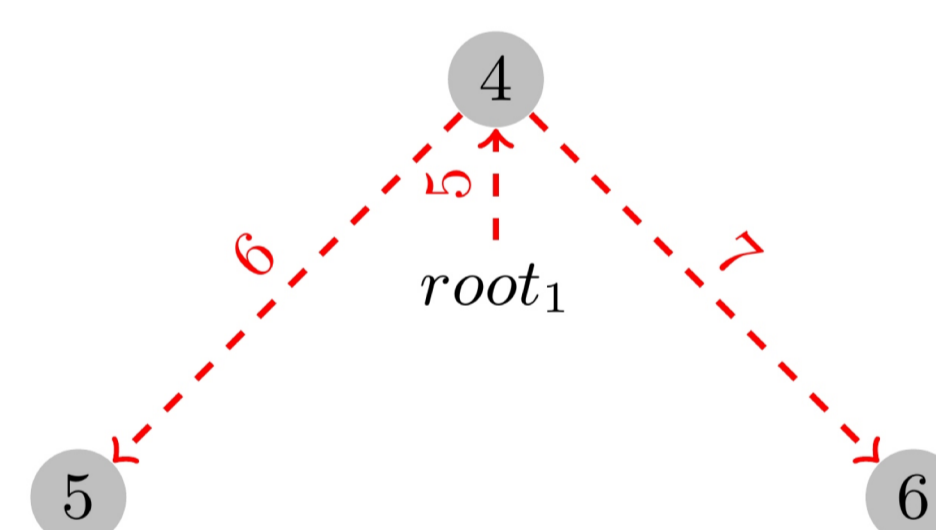
1: procedure FOREST-BP(GRAPH  $G = (V, E)$ )
2:    $T = \{\}$ 
3:   while  $G \setminus T \neq \emptyset$  do
4:      $root \leftarrow \text{random\_select}(V)$ 
5:      $\forall v \in V \Rightarrow \text{transmit}$  beacon  $B$ 
6:     ASSERT: A tree is formed under root because of the Distributed Tree Discovery Algorithm action.
7:      $CNI\text{-Tree-BP}(T_{root})$ 
8:      $T = T \cup T_{root}$ 
    
```



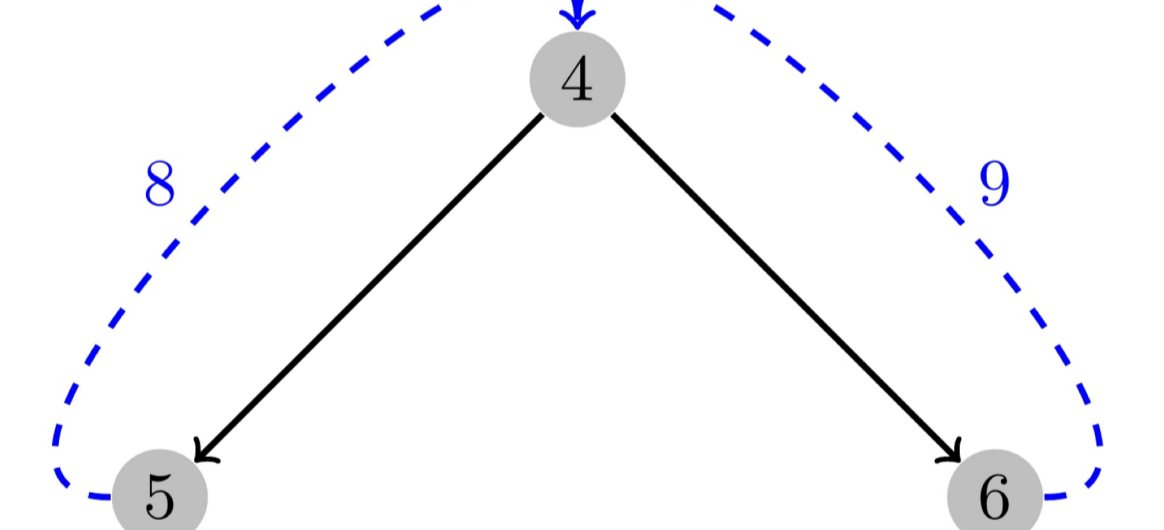
Given a forest  $G$ , randomly select a node to be the first root and form a tree.



Form a backup-placement in the first tree.



Randomly select a new node (not in  $G$  trees) to be the second root and form new tree.



Form a different backup-placement in the second tree.

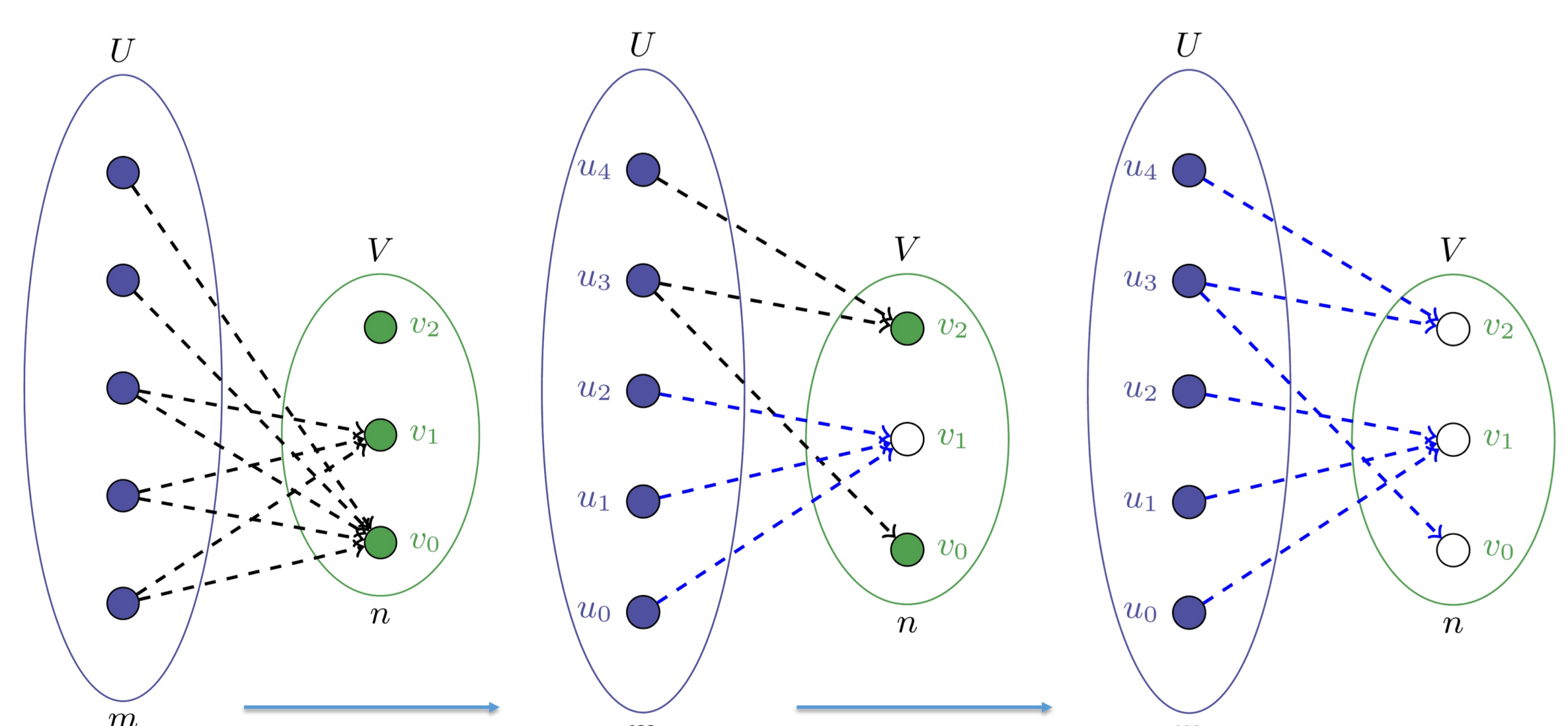
## Backup placement in planar graphs

- We devise a back-up placement algorithm for bipartite graphs  $G = (U, V, E)$ , in which **the maximum degree of vertices in  $U$  is bounded by a parameter  $a$ , and the maximum degree of vertices in  $V$  is unbounded**.
- The goal of our algorithm is obtaining a maximum load of  $O(at)$ . To this end, each vertex of  $V$  may select an arbitrary neighbor in  $U$ . Since each vertex in  $U$  has at most  $a$  neighbors, the maximum load on vertices of  $U$  is going to be at most  $a$  as well.
- We proved that using the Procedure Partition alg' **we reach  $O(\log n)$  time complexity to bipartite and planar graphs** for the backup placement problem.

### Algorithm 5 The Bipartite Graph Distributed Backup Placement Algorithm

```

1: for each  $v \in V$ ,  $v.allow\_backups = \text{Active}$ .
2: procedure BIPARTITE-BP( $V, a, t$ )
3:   if  $deg(v) \in V < 2at$  &  $deg(v) \neq 0$  then
4:      $v.allow\_backups \leftarrow \text{Passive}$ 
5:      $V = V \setminus v, v.neighbors$ 
    
```



**Example of the bipartite graph backup placement**

$t=1$  and  $a=2$ , i.e.  $2ta=4$ , and we allow  $k=2t=2$  backups per vertex in  $V$ .